The lognormal distribution as a market-analytical instrument

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1985

SERIE RESEARCHMEMORANDA

THE LOGNORMAL DISTRIBUTION AS A MARKET-ANALYTICAL INSTRUMENT

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Researchmemorandum 1985-9 Jan. '85

VRIJE UNIVERSITEIT
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Research report on

THE LOGNORMAL DISTRIBUTION

AS A MARKET-ANALYTICAL INSTRUMENT

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The author wishes to thank Dr. P.D. Coljée (project leader of this research project) for his contributions and Prof. Dr. W.J. van de Woestijne and Prof. Dr. A. van Doorn for their comments on the earlier drafts of this report.
He also wishes to thank the remaining members of the Scientific Guidance Committee, Prof. Dr. H.J. Kuhlmeijer, Prof. Dr. R.P. Zuidema and Drs. C.C.J. de Koning for their contributions.

January, 1985.

REPORT OF A STUDY

INTO

THE LOGNORMAL DISTRIBUTION AS A MARKET-ANALYTICAL INSTRUMENT

CONTENTS

1. INTRODUCTION

Regardless of the prevailing market situation it is important to any enterprise to have an insight into the relationship between the price level and the sales potential of its products. Particularly in the present market situation, which is characterised by an extremely wide and varied supply of goods, keen competition among the various types and brands and considerable differences in the purchasing power among consumers, this insight is indispensable for a proper management.
Despite the considerable product differentiation the price-demand relationship (the price-consumption function) is represented in many economic textbooks by an individual sales function (i.e. for a particular enterprise c.q. product). When drawing up a collective sales c.q. demand function many problems have to be faced. In practice there is often a lack of insight into the relationship between the prices and quantities sold of a company's own product and the prices and quantities sold of competing products.
This has led to the desire to develop a different method of describing and explaining the price-quantity combinations as they occur in the current highly complex market situations with their wide and varied supply of commodities.
In doing so, the theory of the lognormal distribution has been chosen. The arguments in favour of this choice are that it involves heterogeneity as its major element, that this theory is widely used in various other disciplines and that it has already provided excellent services in describing a number of phenomena in economics.*)[1)]

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*) Both in natural sciences in general and in biology the lognormal theory is widely used, amongst other examples, in dividing stars according to their size, the distribution of clumps of material after grinding and the distribution of soil particles according to size and weight (cf. Hatch[a)], Hatch & Choate[b)] and Krumbein[c)]). Another application of this theory has been in the growth of organisms and their ultimate distribution according to size (cf. Wicksell[d)] and Cramér[e)]). In economics the lognormal distribution theory has been used to study income distribution and the analysis of consumer behaviour. The research into income distribution was particularly carried out by Kapteyn[f)], Gibrat[g)] and Van der Wijk[h)] whilst Gibrat also studied the distribution of inheritances, bank deposits and individual estates by using the lognormal distribution theory. The analyses of consumer behaviour remained mainly restricted to the income-demand relationship, the so-called Engel curve (cf. Klein[i)] in the analyses of saving behaviour, Farrell[j)] in the demand for automobiles in the U.S., Aitchison[k)] and Aitchison & Brown[l)] on the spending behaviour among the various income groups)[1)].

In the report of this study the provisional results of the research into the possibilities of using the lognormal distribution as a market-analytical instrument are presented.
The study was first of all aimed at using the lognormal distribution theory as a possible description of the price-quantity combinations as they occur in the present market conditions with their extremely wide and varied supply of heterogenous commodities.
The results that have been obtained agree, amongst others, with the results obtained by Schultz and those achieved by Adam. Schultz postulated, in 1938, a lognormal relation between the prices and the demanded quantities of goods [2]. Adam studied in laboratory situations the pattern of the consumers' preferences. In doing so, he arrived at the conclusion that the differences among the consumers' preferences as expressed in their willingness to pay a particular price for a particular product, are such that one may speak of a lognormal distribution [3]. Also in the case of empirical market investigations into the sales of products that are only bought once during a long period of time (i.e. durable consumer goods e.g. refrigerators and washing machines) Adam was able to prove the existence of a lognormal relationship between the price and the quantities sold.
This means that, in practice, simultaneous price differences may occur at incompletely separated (sub-)markets for the same product or a comparable product. Such differences in appreciation can be observed when the product is looked upon as constituting 'a bundle of characteristics/services' in which the 'characteristics' of the distribution channel are also included in the 'chacteristics of the product'. Not only do the products show mutual differences as far as their characteristics are concerned but also the buyers' valuations of these characteristics show differences. The effect of using market instruments, such as price, quality, sales promotion and distribution is therefore of major importance to the consumers' valuation of a particular product.

The report of this study will successively deal with the following aspects: A brief description of the theory of the lognormal distribution followed by the testing method and the testing results. This is followed by an illustration of the potential application of this theory as a market-analytical instrument based on a product of which there is a fixed supply (i.e. hotel rooms), and some non-durable consumer goods.
The object is not only to arrive at an analysis of one particular market situation but also to compare a number of successive market situations.

II THE LOGNORMAL DISTRIBUTION THEORY

The lognormal distribution theory consists of three essential components i.e. the stimuli (influencing factors), the subject and the response. When these components are introduced into the theory of consumer behaviour the following picture is obtained.

Each potential buyer is affected by a great number of stimuli, such as the product price, quality, advertising, distribution channel, income, changes in income, social background, educational level and experience. Each positive response from a subject is expressed in the form of a purchase. The remaining important element is then what kind of purchase is made i.e. the quantity of products purchased and the price that is paid.

In fact, when buying a product every consumer makes a number of simultaneous and essential decisions. When arranged in hierarchical order these decisions could be stated as follows: 1. The decision whether or not to buy the product. 2. The quantity to be purchased. 3. The price the buyer is willing to pay and the quality he selects.[4)]

According to the Central Limit Theorem a 'normal' distribution (the Gauss or probability distribution) of the responses is achieved when these decisions are caused under the influence of a great number of additional, independent and random factors. In this case the individual factors may either have a positive or a negative influence, or no influence at all. The sum of these factors provides in each individual example the intensity value of the total number of stimuli and, consequently, of the ultimate response.

Since economic variables, such as prices, incomes etc. are by their very nature restricted to positive values only, the 'normal' distribution pattern is not applicable to them. The simplest distribution model analogous to the 'normal' distribution, which excludes the negative and zero values of the variables, is the 'lognormal' distribution. ('skew Gauss distribution').

This distribution occurs when a great number of independent, random factors with a multiplicative effect exert a positive influence.[5)] The multiplication product of the separate influencing factors is then decisive for the intensity value of all the stimuli and, consequently, for the value of the response. The responses displayed by the buyers will therefore show proportional differences[*)].

Generally speaking, the buyers' responses, as expressed in their purchases of goods at particular prices (due to the effect of a very large number of multiplicative, positive, independent and random factors) will show a lognormal distribution.

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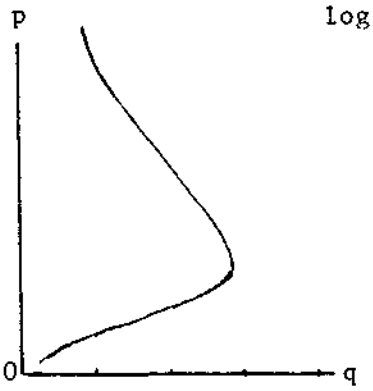
*) This is in agreement with the Weber-Fecher Law[6)] which states that in human actions the response is proportional to the logarithm of the stimuli. Since the logarithm of the responses complies, in turn, with the 'normal' (i.e. additive) model, this may be called the 'lognormal distribution' or the skew Gauss distribution.

The form of the lognormal frequency distribution or skew Gauss distribution is displayed in Fig 1. Actually, this concerns a probability distribution of the responses of the buyers expressed in prices (p) and quantities (q). If the logarithm of the price is plotted on the price axis and the percentage of the total quantity in each price (class) on the quantity axis, a distribution of the 'normal' type, a density function (Fig 2) is obtained. In order to gain a better insight into the distribution of the anticipated responses across the overall market size, this function has to be integrated. Thus the integral of the skew Gauss distribution, the lognormal distribution (a distribution function, Fig 3) is arrived at. This integral forms the basis of a market analysis by the lognormal distribution.



Quantity (q) per price class

Figure 1

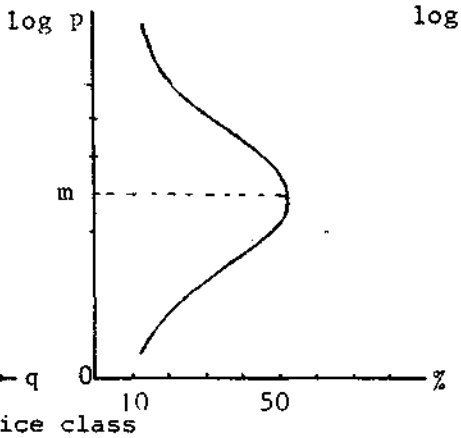


Figure 2

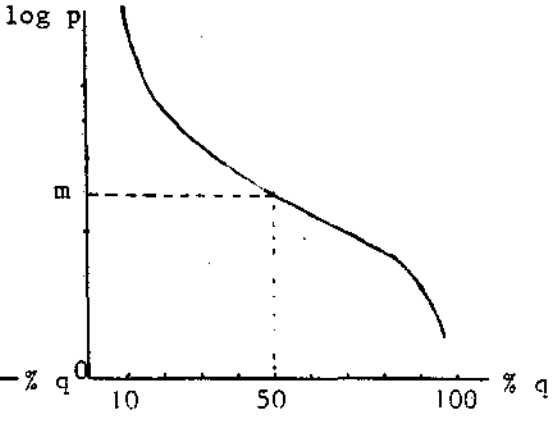


Figure 3

This integral shows that, if, in a practical example, a great many independent, positive, random and multiplicative factors make their influence felt, the number of transactions (quantity of goods) that are closed at the various prices can be expected to show this integral pattern.

It would seem highly likely that a great number of factors affect the buyers' activities in buying situations, having regard to the examples given (i.e. price, quality, sales promotion, income etc.).
However, the multiplicative effect of these factors still requires some explanation. The normal distribution is used more frequently than the skew distribution, which might create the impression that additivity is far more general than multiplicativity. However, one of the conditions for additivity is that the factors must be of the same dimension.
The same restriction as regards their effect does not apply to multiplicativity which would therefore seem to be more general than additivity. This means that the normal distribution could actually be regarded as a special case.
In the foregoing it was already pointed out that a potential buyer makes a number of simultaneous (as against successive) essential decisions when closing a transaction.

In these simultaneous decisions made by the consumer the multiplicative effect of influencing factors is highly probable.
The effect of such influencing factors is expressed when a transaction is closed at a particular price (i.e. the subjective valuation of a transaction). Research has proved that the subjective valuations of buyers as expressed in their willingness to pay a particular price for a product show a proportional relationship.[7] This proportional relationship between the responses is only possible when the influencing factors exert a multiplicative effect.

Starting out from this assumption Van de Woestijne developed, in 1953, a general demand theory based on the lognormal distribution theory[8]. The starting point for his argument was that the decision to close a transaction, i.e. the purchase of a unit of a particular product, is affected by a great number of multiplicative factors. Each of the individual factors has no noticeable, i.e. measurable, effect of its own on the ultimate result, i.e. closing the transaction. No single transaction is identical to another. All transactions differ as to their object, i.e. the product involved, the subjects involved, i.e. the buyer and seller, the location of the action, i.e. the market/distribution channel, the time of action and the remaining influencing factors, e.g. influence of the environment. Even two successively occurring transactions by the same individual relating to an identical product occurring in the same location are not quite identical to each other.
This completely heterogeneous collection of transactions forms the starting point for the earlier-mentioned theory of demand. Based on the available assumptions the distribution of the transactions according to prices and quantities will be a lognormal distribution by definition.
Depending on the nature of the problem to be investigated the heterogeneity of this completely heterogeneous collection of transactions can be reduced to object, location or time of the action.
As long as this restriction in heterogeneity is not combined with the introduction c.q. manifestation of important (systematic or non-random) factors; the collection of these transactions will remain lognormally distributed. In particular the deviations observed in the anticipated lognormal course of the selected collection of transations are important when using the lognormal distribution as a market-analytical instrument.
Deviations from the expected pattern may be said to exist when there are circumstances which make it impossible for buyers actually to express their responses to the stimuli according to the normal expectation along the lognormale distribution pattern. Globally speaking, the following causes of such deviations may be mentioned:

- Reduction in homogeneity : In selecting the product category, product group or products, which will form the subject of the study, products have been joined which, in the eyes of the buyers, do not belong to the same category/group, e.g. showing considerable substitution-discontinuity or difference in competition intensity.

- Supply : The supply of the product is insufficiently large and varied in quality, price (price classes) and quantities per price (class).

- Distribution channels : The number of distribution channels is insufficiently large and varied in quality, shop profile, price image, potential service or geographical distribution.

- Buyers : There is no sufficiently large heterogeneous group of buyers. The group of buyers consists of two groups which can be distinguished on account of different characteristics. Within the group of buyers considerable valuation differences are to be found in respect of one or more selected products from a collection (cf. reduction in heterogeneity).

- External factors : These factors may affect the possibility of concluding transactions according to the anticipated pattern to such a considerable extent that the influence of an important c.q. systematic factor may be said to be at work. As examples may be mentioned: government steps, such as minimum/maximum price regulations, rationing, subsidies, taxation and other interference in the price and market mechanism. In addition, other environmental factors may also play a major part e.g. the influence caused by institutional operations or the weather.

The possible causes which are responsible for deviations in the anticipated lognormal distribution of the transactions will be discussed in further detail in the final part of this report.

III TESTING THE DISTRIBUTIONS FOR THEIR LOGNORMAL PATTERN

1. Testing method

Within the scope of the study into the possible uses of the lognormal distribution as a market analytical instrument no testing based on purely statistical/mathematical techniques of distributions for their lognormal pattern has taken place. Tests of this kind require an extremely comprehensive and time-consuming algebraic/mathematical treatment of the figures concerned. In order to achieve a statistically/mathematically justified test the unprocessed research data have to be tested on the pattern of the lognormal frequency distribution (density function - Fig. 1). For the actual purpose of the present research, i.e. testing whether or not the observed price-quantity combinations are lognormally distributed, such an investigation can, in our opinion, be dispensed with. Since the present study has as its particular object the conspicuous, eye-catching deviations from a lognormally distributed pattern, a less exact testing method should be adequate.
This test, which actually amounts to a plausibility test, is based on the use of lognormal graph paper. This technical paper shows a logarithmical distribution on the price axis and a probability distribution on the quantity axis. A special characteristic of this paper is that lognormal distributions, i.e. the integrated density function or distribution function, Fig. 3, are represented as straight lines.

In order to perform this plausibility test with regard to the lognormal pattern of a distribution, data are required concerning the number of transactions closed in a particular product c.q. product group at the various prices (price classes). These data can be obtained from a random sample investigation, panel investigation or direct measuring.
The prices or price classes are arranged from high to low. The quantities at each price (class) are expressed as a percentage of the total quantity sold. This percentage is then cumulated per price class. An example of this processing of the investigation data is given in Table I. This concerns a fictitious example of price-quantity combinations for product A on market N during period P which were sold at widely varying prices.

TABLE I

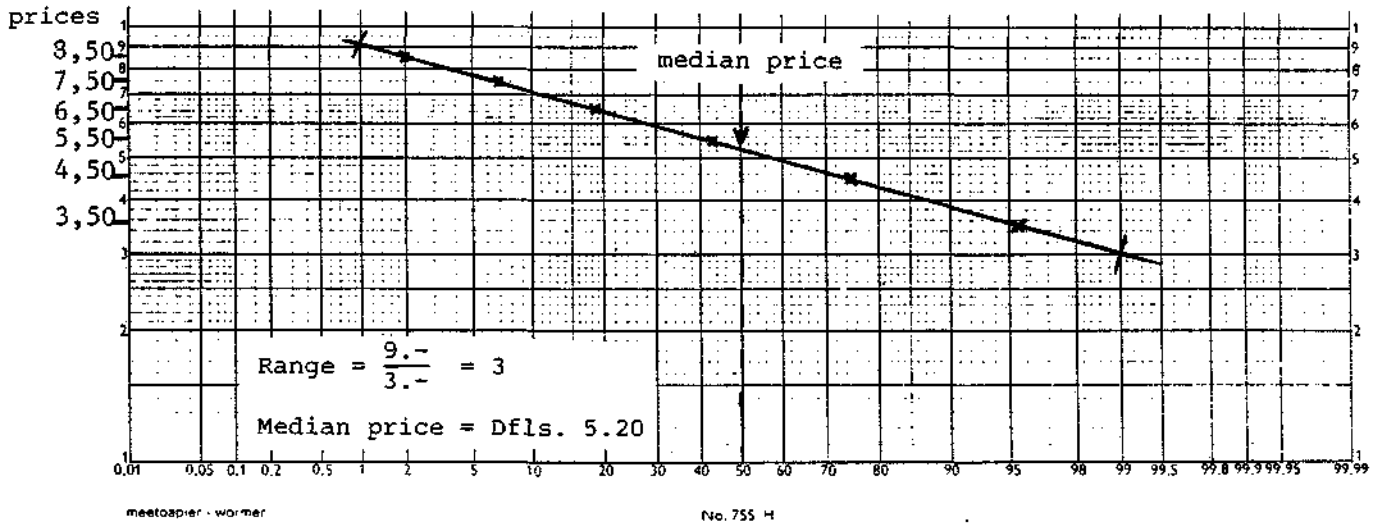
SALES OF PRODUCT A ON MARKET N DURING PERIOD P

Price class product A	No. of units	Quantity as a percentage of total quantity	Cumulated percentage of the total quantity
Dfls.8.50 and over	72	2.0	2.0
Dfls.7.50 - 8.49	180	5.0	7.0
Dfls.6.50 - 7.49	414	11.5	18.5
Dfls.5.50 - 6.49	882	24.5	43.0
Dfls.4.50 - 5.49	1,152	32.0	75.0
Dfls.3.50 - 4.49	738	20.5	95.5
Dfls.3.50 and below	162	4.5	100.0
	3,600	100.0	

After arranging the prices (price classes) from high to low with the corresponding quantities, calculating the percentage of the overall quantity per price class and then cumulating these percentages at each price class, the calculated price-cumulated quantity percentages can then be plotted on the lognormal paper.
For this purpose the lower price from each price class and the corresponding cumulative quantity percentage is taken. This shows, for example, that 43% of the total quantity was sold at a price of Dfls. 5.50 or more. These price-quantity combinations are then plotted on the lognormal paper, which, in the present instance, produces 6 observation points, the lower price class of Dfls. 3.50 and less being deleted. Within the scope of this study the line connecting the observation points is called the transaction curve. The transaction curve for the figures of the example is shown in Fig. 4.

Fig. 4

TRANSACTION CURVE SHOWING THE SALES OF PRODUCT A



Cumulative quantity percentage

Before proceeding to deal with the major key values of the transaction curve and the construction method a few remarks should be made in regard to the basic material.
As appears from Table I this (fictitious) product is offered at prices varying from Dfls. 3.50 and less to Dfls. 8.50 and more at the same time c.q. during a brief period. On the overall market for this product, i.e. the total of all the distribution channels, there is therefore no question of price unity. The total market is actually divided into a great number of sub-markets that are not strictly separated and consisting of the various distribution channels. These distribution channels differ in quality, price image, shop profile, service potential etc. The geographic distances separating certain distribution channels and the personal preferences of the buyers also play a part in the division of these sub-markets. This means that it is, in practice, possible that very widely differing prices are demanded for the same product at different sales outlets which may be quite adjacent to each other. The following examples may serve as an illustration: suntan lotion - being at least twice as expensive along the coast as further inland, radio and TV equipment sold by a cash-and-carry trader or by the professional dealer with his extensive after-sales service and food by the various supermarkets or the small local shop around the corner. Even when the consumer is aware of the existing price differences he is not always prepared to travel to the sales outlet offering the lowest price on account of the cost of information, transport (transactional costs) or even his personal preference. Identical prices are naturally offered simultaneously within each sub-market (distribution channels).
The table shows that the quanty of products demanded declined when the price dropped from Dfls. 4.50 to Dfls. 3.50 and less. Although these figures are fictitious, such reductions in sales at lower prices often occur in practice. In view of the pattern of the price-consumption curve a rise in sales at lower prices would seem to be more likely to expect. For inferior goods such a drop in sales at lower prices can very simply be explained. But the number of inferior products in relation to the income is generally considered to be very small. In other words, a different explanation for this decline in sales has to be offered.
In these examples there may be a question of market saturation which may occur for each product. The relatively declining total demand, i.e. the aggregate demand, cf. Fig. 3, beyond a particular point is the result of the decline of the marginal demand (below the backward bend of the skew Gauss distribution, cf. Fig. 1). This relative decline in the total demand upon the approaching market saturation is in complete agreement with the expectation pattern based on the lognormal distribution theory.
Market saturation for a particular product or for an entire products group may be caused by various factors.

Among other causes may be mentioned: declining confidence among customers in the quality-price relationship, since buyers operate within certain price limits. This is often correct, since below a certain price an economic production/supply of the product is no longer justified. Another reason may be that a large supply of the product concerned is already available to the buyers so that a reduction in price no longer results in increased sales.

Having made these remarks on the basic research data which are often found in practice the criteria for the construction and the key values of the lognormal distribution will now be discussed.

In the above example it was clear that the line connecting the various transaction points is a straight line on the lognormal graph paper (Fig. 4). In this case there is a completely lognormal distribution of the number of transactions within the various price classes.

Generally speaking, however, there is no question of encountering such a perfectly straight line as was shown by this example. The connecting line of the points of observation, i.e. the transaction curve, then often displays a more or less irregular pattern for certain parts of the market size. In order to trace any deviations from a completely lognormal course of the transactions it is important that the most qualified lognormal distribution should be determined as a frame of reference, which is called the normline. Visually it is possible to draw a straight line which provides generally reliable estimates as regards the parameters according to Cramer[9]. With the help of this line, i.e. the normline, which is thus constructed a rapid check on the lognormal pattern may be carried out[10].
Among other criteria for the construction of this straight line connecting the points of observation the following may be mentioned:
- the number of observation points situated on, or approximated very closely by the straight line
- the market size in respect of which this straight line directly connects the points of observation or approaches these points most closely.

It should be noted that the probability scale is particularly sensitive to small deviations approaching 0% and 100%. This means, as Cramer remarks, that the deviations near the very high and very low prices should not have too much influence on the construction of this line.[11]. The sensitivity of the probability scale is not the only cause of the incidence of deviations near the very high and very low prices. Another reason for this is that relatively few transactions are, generally speaking, concluded at these prices so that only few observations can be made in regard to these high price and low price transactions.

As key values for a lognormal distribution in this study will be used the range and the median price. A distribution is completely determined and registered by means of these two values, which is a necessary requirement when comparing distributions. The range (R) is the proportion between the price at a market size of 1% (p_h) and the price at a market size of 99% (p_l) i.e.

$$R = \frac{p_h}{p_l} .$$

It is also possible to fix ranges at other percentages of the market size, such as 5% and 95% or 10% and 90%. However, in this study the proportion between 1% and 99% will be used. From the graph the visually drawn straight connecting line between (or approaching) the transaction points, the prices can be read at a market size of 1%, i.e. in this case Dfls. 9.-, and 99%, i.e. in this example Dfls. 3.-. This means that the range in this example will be Dfls. 9.- : Dfls. 3.- = 3. The median price is the price at a market size of 50% (p_m), i.e. in this example Dfls. 5.20, which can also be read from the graph. By definition the median price of a log distribution is the mean between the highest and the lowest price. The relation between the range and the median price can be stated as $p_h : p_m = p_m : p_l$. Since this range has been defined as:

$\frac{p_h}{p_l}$, it may be stated that $p_h = p_m . \sqrt{R}$ and that $p_l = p_m \cdot \frac{1}{\sqrt{R}}$.

By means of these formulae a check can be made to ascertain whether the prices read from the graph are sufficiently accurate.
When these formulae are applied to the example that was given, the result will be:
$p_m = p_h : \sqrt{R}$ or $p_l \cdot \sqrt{R}$. Completing the equation results in:

$p_m = 3. \sqrt{3} = 3. 1.7321 = 5.1963$, which corresponds with the value that can be read from the graph for the median price of Dfls. 5.20. This means that the values read from the graph and the median price are sufficiently accurate and that they agree with one another.

The above testing method to check whether or not the pattern of the number of transactions is lognormal at the various prices therefore constitutes a plausibility test performed by means of lognormal graph paper. The investigation results are arranged and plotted on the lognormal paper before the points are connected into the transaction curve. Based on the observation points a straight line, the normline, is then drawn who course either intersects or approaches the observation points very closely, taking into account the market size in question. In determining the range and median price of the normline the latter is fully determined and registered.
If all the observation points are situated on this normline, the expectations as to the lognormal distribution course of the demand are fully met. The normline and the transaction curve coincide. If the transaction curve deviates from the normline, the expectation as to the lognormal distribution of the transactions only remains justified if plausible reasons may be advanced to explain the nature and size of such deviations. The results of the above test of the lognormal distribution pattern of the demand will be discussed in the next section.

2. Results of the test

The plausibility test of the theory of the lognormal distribution of the demand took place, apart from other figures, with statistical data obtained from the market research agency, Attwood B.V. of Dongen, The Netherlands. This research agency has a consumers panel of 2000 - 5000 persons available who record their purchases weekly according to the products and brands bought, prices paid and the quantities and locations where the purchases took place.
In addition to testing on the basis of these panel data, also other data gathered from random sampling and direct measuring were obtained. Although statistians differ in their opinion as to the reliability of study results obtained on the basis of the panel method on the one hand and the random sampling method on the other, it should be mentioned, even at this stage, that as far as the test of the lognormal model is concerned the possible differences in reliability between the two investigation data were proved to be irrelevant to the plausibility test that was used. All the data used in the study yielded comparable results within the scope of this study.

Based on the processing and testing of the data of approximately 50 different consumer goods divided into 12 different product groups the expectation of a lognormal probability distribution of the transactions at various prices has become very plausible.
The test dealt with individual products and product groups that were offered on the Dutch market. The data were also split up to a maximum extent according to the various distribution channels.

Generally speaking, a lognormal distribution was found of the numbers of transactions that were concluded at various prices (price classes). If any deviations did occur, they were often found to occur at extremely high or low prices where certain reservations have been made as to the sensitivity of the probability scale at these values. No special attention was therefore required for these deviations, also having regard to the small number of observations. The remaining deviations that were found were either relatively small in size so that they were not relevant in this plausibility test (being insufficiently conspicuous), or, in other instances, they could be explained by the incidence or manifestation of important, c.q. systematic or non-random, factors. Based on these provisional test results the lognormal distribution was then used as a market-analytical instrument in which the deviations from the anticipated lognormal distribution form the basis of the analysis. When a market shows a sufficiently large and varied supply of products, qualities, prices, quantities as well as a sufficient variety in the disbrition channels, i.e. the selling locations, and the buyers form a sufficiently large heterogeneous group without any varying characteristics among the sub-groups, the distribution of the number of transactions concluded at various prices will be lognormal (skew Gauss distribution). On lognormal graph paper this distribution will be approximately represented by a straight line.

3. Possible interpretation problems during the test.

The number of transactions concluded for a certain product at a certain price (price class) will generally be determined by the size of the demand for this product in this price (class). In practice, three different market situations, i.e. supply/demand relationships, can be distinquished which may be decisive for the number of transactions that can be concluded at the various prices (including stock mutations) i.e.:

1) The quantity offered is larger than the quantity demanded at a given price
2) The quantity offered is equal to the quantity demanded at a given price
3) The quantity offered is smaller than the quantity demanded at a given price.

In the first example the quantity demanded is the restricting factor as far as the number of transactions concluded is concerned. In the second example both the quantity offered and the quantity demanded form the restricting factors whilst only in the third example the restricting factor is formed by the quantity offered. Generally speaking, however, experience has shown that the quantity offered for each price (class) is large enough to meet the demand. This means that the observed price-quantity combinations represent the points of a demand curve in the large majority of cases.*)

The question whether all the observed points are really part of one and the same demand curve is described in literature as the identification problem. For there may be a shift in the demand curve so that several intersections occur with the same or another equally shifted demand curve. A connecting line between such points does not in this case produce the desired demand curve but a regression line. The demand curve can only be determined when the observed points are intersections of one and the same demand curve with various other (original or shifted) supply curves.

In the foregoing it was concluded that the subjective valuations of a sufficiently large heterogeneous group of buyers, inasmuch as such valuations are expressed in the prices paid, show a logarithmic pattern. When various demand curves occur, this will also produce various logarithmic distributions. If the buyers group consists of two or more segments that may be distinguished on account of essential differences in their characteristic attributes, the subjective valuations of the entire group expressed in prices can no longer be represented in a single logarithmic distribution. Each sub-group will show its own characteristic valuation course which is expressed in a specific logarithmic distribution for each group. This means that as long as the observed price-quantity combinations are approximately situated on one straight line there is no question of any identification problem and that the most likely underlying demand curve has been established.

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*) By demand curve is in this case understood the collection of price-quantity combinations which might evolve on account of the buyers' preference for a particular product in a particular period.

IV ANALYSIS OF A MARKET SITUATION BASED ON THE LOGNORMAL DISTRIBUTION

1. Establishing the product range and prices for a product group

To illustrate a market analysis in a particular situation the example of establishing the prices of hotel rooms with various qualities will be used as described by Kent B. Monroe in his book 'Pricing'[12]. This example has been chosen in particular, since a hotel may be seen as a product group of rooms with different qualities and prices so that the distribution of the supply can be established very easily. In practice, it is very difficult to assess the supply when analysing the market for other products, since an elaborate investigation is required for this.
The hotel is an old-fashioned hotel with 600 rooms showing a considerable difference in quality. The hotel is using 21 different prices for its various rooms. In order to establish the supply Monroe determines the number of rooms that are offered at a particular price before expressing this number as a percentage of the total of 600 rooms. The demand for these hotel rooms was determined by registering the rent of the hotel rooms at the varous prices during two periods of 10 days, taking into account an average occupancy rate of 75% which was normal for this hotel. The number of rooms that was demanded at the various prices was then expressed as a percentage of the total demand for these rooms. After ranking the hotel room prices from high to low the demand and supply percentages for each price were then cumulated. The results are shown in Table II.

In order to explain the possibilities offered by the analysis based on the lognormal distribution we will first take the analysis by Monroe so that the subsequent application of the lognormal method will clearly show up the shortcomings of the method used by Monroe.
When the supply and demand for these rooms at the various prices had been established, the rooms were examined and criteria for the noticeable differences in quality were determined. Each room was then examined for the number of noticeable attribute differences (view, room size, TV, air-conditioning, floor, refrigerator etc.) and a room classification was drawn up. It was then concluded that the number of prices used was considerably greater than the number of classifications established based on the perceivable quality criteria. The number of determined quality classes was 9.
Starting out from the usual price differences for this hotel the price for the hotel rooms in the lowest quality classification was maintained at $ 16.- whilst the rooms in the highest quality classification were assigned the highest price that had been used so far, i.e. $ 35.-.

TABLE II

Hotel room occupancy and hotel room supply according to Kent B. Monroe.

Hotel room price in dollars ($)	Number of hotel rooms	Supply percentage	Demand percentage	Cumulative percentage Supply	Demand
35.00	10	1.7	0.5	1.7	0.5
32.00	15	2.5	1.0	4.2	1.5
30.00	5	0.8	0.5	5.0	2.0
29.50	5	0.8	1.0	5.8	3.0
29.00	10	1.7	1.0	7.5	4.0
28.00	5	0.8	0.5	8.3	4.5
27.50	5	0.8	0.5	9.1	5.0
27.00	10	1.7	1.0	10.8	6.0
26.00	5	0.8	1.0	11.6	7.0
24.00	25	4.2	3.0	15.8	10.0
22.00	10	1.7	5.0	17.5	15.0
21.00	10	1.7	7.0	19.2	22.0
20.00	20	3.3	8.0	22.5	30.0
19.50	10	1.7	10.0	24.2	40.0
19.00	15	2.5	20.0	26.7	60.0
18.50	40	6.7	10.0	33.4	70.0
18.00	20	3.3	10.0	36.7	80.0
17.50	55	9.2	8.0	45.9	88.0
17.00	285	47.5	3.0	93.4	91.0
16.50	20	3.3	4.0	96.7	95.0
16.00	20	3.3	5.0	100.0	100.0
	600	100.0	100.0		

In order to determine the intermediate prices Monroe employs a method based on the Weber-Fechner law[13] (the response being proportional to the stimuli cf. p. 3). According to this method the stimuli, in this example being the price differences, should be large enough for buyers to be observed. The difference between each next pair of prices (price classes) can now be determined by means of the logarithm of the prices, since price differences are not perceived as an absolute but as a relative factor.
This formula, which was developed by Monroe based on the Weber-Fechner law, is then found to be

$$\text{Log } k = \frac{\log 35 - \log 16}{n - 1}$$

in which k is the factor by which each of the prices has to be multiplied in order to establish the next price and n is the number of price classes, i.e. in this case 9. When we calculate the answer we find that k = 1.103. The lowest price class of $ 16.- is then followed by the price 1.103 x $ 16.- = 17.648 rounded to $ 17.65, which is later rounded to the nearest multiple of $ 0.25 i.e. $ 17.75.

Monroe then matches the hotel room supply at each price to the demand at that price in which he ignores any possible special factors that may have affected the demand schedule that was found. Table II clearly shows that the original supply was particularly badly adapted to the demand. In the higher price classes, i. e. exceeding $ 27.-, the supply percentage exceeds the demand percentage while the cumulative supply percentage already exceeds the demand percentages from $ 22.- upwards.
The greatest difference between the demand and supply percentages occurs at the price of $ 17.50. For at this price or higher prices 45.9% of the rooms are offered whilst 88.0% of the total demand occurs at this price or higher prices (showing a difference of 42.1%). Without further investigating the possible influence of the excessively small supply at certain prices on the demand, Monroe then establishes the supply percentages based on the determined demand. If 5% of the total number of rooms is demanded at a given price, he believes that also 5% of the total number of rooms should be offered at this price, i.e. 30 rooms. In this way he obtains a survey of prices which have been adapted to the perceivable quality differences between the various rooms and the number of rooms that should be provided at each price based on the demand percentage.
The results which are thus obtained are shown in Table III.

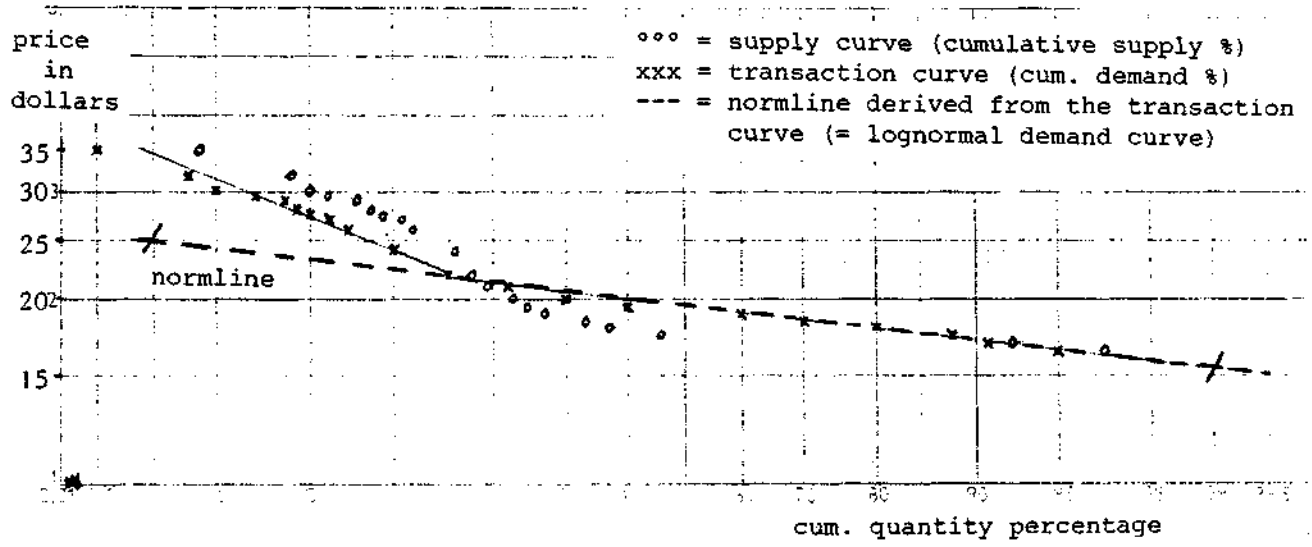
TABLE III

Improved pricing and adaptation of the hotel room supply based on the determined demand according to Monroe.

Room price in dollars		Calculated no.	Percentage of	Cumulative
Calculated	Rounded to $0.25	of hotel rooms	total number (600)	percentage total
35.05	35.00	5	0.83	0.83
31.78	31.75	10	1.67	2.50
28.81	28.75	15	2.50	5.00
26.12	26.00	30	5.00	10.00
23.68	23.75	90	15.00	25.00
21.47	21.50	120	20.00	45.00
19.47	19.50	150	25.00	70.00
17.65	17.75	125	20.83	90.83
16.00	16.00	55	9.17	100.00
		600	100.00	

Following this discussion of Monroe's method of pricing, we will now explain the pricing method by means of the lognormal method.
For this purpose the data from Table II, i.e. the cumulative supply and demand percentages are first of all plotted on lognormal graph paper. The results are shown in Fig. 5.

FIG. 5
Distribution of the original supply and demand percentages of hotel rooms at the various prices, according to Kent B. Monroe

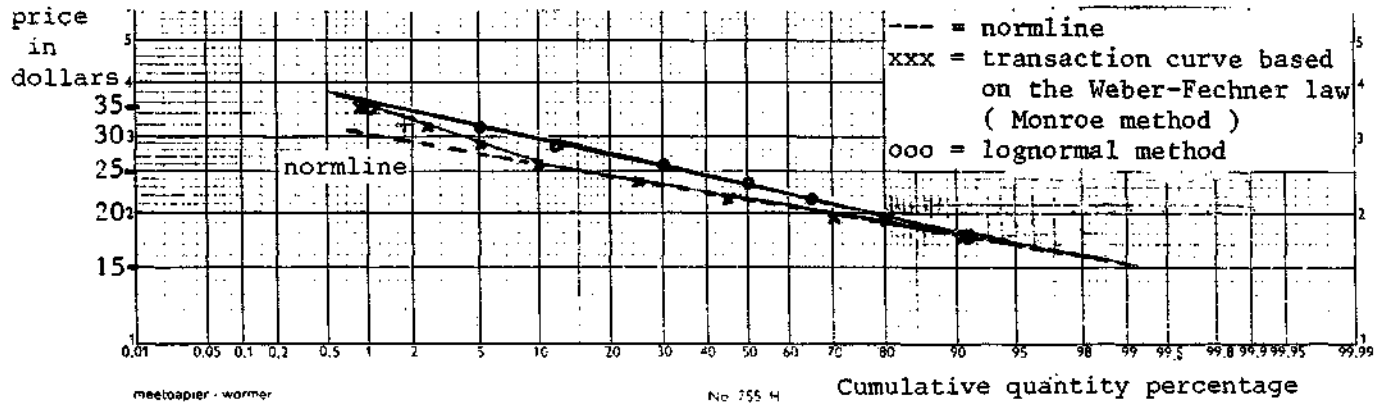


The connecting line of the transactions actually closed at each price forms the transaction curve (xxx in Fig 5).
The distribution of the supply throughout the various price classes is shown by the supply curve (ooo in the graph, not continuous).
The majority of the transactions concluded, i.e. between 15 - 95% = 80% of the total number, shows a completely lognormally distributed pattern of the transaction curve. By drawing a straight line connecting a great number of observation points which cover a large part of the market volume a lognormal distribution (normline) is obtained which would have occurred without the incidence of one or more important (systematic or non-random) factors.
With the help of the normline, which is thus constructed, deviations from the basic lognormal distribution become clearly visible. In this example a deviation is seen in the first 15% of the market volume. In the analysis based on the lognormal distribution this kind of deviation forms a direct reason for a more detailed investigation into the factors which may account for this deviation.
A survey of potentially important factors was given on page 6. In the interest of the illustrative value of this example no attention will be paid to the nature and composition of the demand, or external factors as the possible cause of the incidence of this deviation, one of the reasons being that an investigation into the demand or into external factors has become almost impossible. However, even without this assumption it is highly plausible that the cause of this deviation has to be looked for in the supply, having regard to the course of the supply curve and the deviation concerned.

For each of the transaction points covering the first 15% of the market volume, i.e. the lognormally deviating part of the transaction curve compared with the normline, the demand percentage at each price (class) may be said to remain considerably behind the supply percentage. As a result, it may be concluded that for a certain percentage of the market volume the price at which the rooms are offered is considerably higher than the price the guests are prepared to pay.
In regard to the remaining part of the market volume, i.e. 15% - 99%, to which the course of the normline relates, it may be said that the supply percentages at each price (class) are considerably below the demand percentages, in other words, for a certain percentage of the market volume the price at which the rooms are offered is clearly lower than the price the guests would have been prepared to pay.
If the supply is to be adjusted to the demand, either the prices for one part of the market (0 - 15%) have to be reduced or the number of rooms offered at each price (class) have to be reduced.
For the remaining part of the market either the room prices have to be increased or the number of rooms offered in each price class has to be increased. However, all this will have to take into account the lognormally distributed demand which establishes the relationship between the quantity demanded and the price level.

The pricing method used by Monroe in this analysis is solely based on an adjustment in the prices and the distribution of the supply based on the demand percentages found. In doing so, the possible influences on the demand caused by an 'imbalance' on the supply side are completely ignored. If the results of this improvement by Monroe are plotted on lognormal graph paper, it turns out that a deviation continues to exist for the first part of the market (10%). Although this deviation is smaller, both as regards the market volume and the slope (angle) with respect to the normline compared with the initial situation, the matching of the supply to the demand does not prove to be optimal (cf. xxx in Fig 6).

FIG. 6
Improvement in the pricing and the supply and demand percentages based on the Weber-Fechner law (Monroe) and on the lognormal method



Starting out from the same prices and price differences as calculated by Monroe, the lognormal distribution method produces a different classification of the rooms at the various prices. For this purpose the cumulative percentages at each price are read from the graph (cf. ooo in Fig 6) and converted into percentages and numbers of rooms. The result is shown in Table IV.

TABLE IV.

Pricing based on the lognormal method

Room price in dollars	Cumulative percentage	Percentage total number of rooms (600)	Calculated number of hotel rooms
35.00	1.0	1.0	6
31.75	5.0	4.0	24
28.75	12.0	7.0	42
26.00	30.0	18.0	108
23.75	50.0	20.0	120
21.50	65.0	15.0	90
19.50	80.0	15.0	90
17.75	91.0	11.0	66
16.00	100.0	9.0	54
		100.0	600

In fact, when pricing the hotel rooms one may start out from two different lognormal distributions. If we start out from the original situation an obvious step would be to carry out the pricing based on the straight line drawn through the greatest number of observation points covering a large part of the market volume (the normline in Fig 5). However, as mentioned earlier, as far as the major part of the market volume is concerned, the prices at which the rooms are offered lag considerably behind the prices the guests are prepared to pay, as is evident from the course of the normline.

Actually, Monroe arrives at the same conclusion, since the normline that can be drawn based on his improvement shows a greater range (slope) than the one in Fig. 5 (cf. Fig. 6).

When pricing on the basis of the normline drawn according to Monroe's improved method (see Fig 6), the prices could show a maximum variation of between $ 16.- and $ 30.-. This does not agree with the original object which was that the original maximum price of $ 35.- and the minimum price of $ 16.- should be maintained. The starting point of the lognormal improvement that has been applied is therefore not formed by this normline but by the lognormal straight line above it (ooo in Fig 6). In doing so, the prices established by Monroe are maintained.

However, the number of rooms to be offered at each price has been determined on the basis of the assumed lognormal pattern of the demand. By making use of the higher line of the lognormal distribution the trading results will be considerably improved compared with Monroe's improvement (cf. Table V). Using this method also reduces the chances of the customers receiving a 'consumer premium', which would be the case if the demanded prices were to be lower than those customers are prepared to pay.

Assuming that the running expenses of the hotel remain the same for each of the three pricing methods, a comparison of the results produced by each of the variants can be made.
The average room occupancy in this hotel is 75% (cf. p. 14). This means that an average of 75% of the hotel rooms offered in each price class is occupied, in so far as the supply is matched to the demand.
A calculation of the revenues produced by each of the three methods will therefore be based on the demand percentage. Out of the total number of rooms, i.e. 600, there is an average demand for 75%, i.e. 450 rooms. This means that 1% of the demand percentage equals 4.5 rooms. The number of rooms is then multiplied by the room price. The revenues obtained in this manner are shown in Table V.

TABLE V

Calculating the hotel revenues for the three different pricing methods

Room prices in dollars		Demand percentage and attendant number of rooms based on 75%						Revenue in dollars in case of:		
original	based on Weber-Fechner law	original %	original number	Monroe %	Monroe number	Lognormal %	Lognormal number	original	Monroe	Lognormal
35.00	35.00	0.5	2.25	0.83	3.37[5]	1.0	4.50	78.75	130.72[5]	157.50
32.00		1.0	4.50					144.00		
	31.75			1.67	7.51[5]	4.0	18.00		238.60	571.50
30.00		0.5	2.25					67.50		
29.50		1.0	4.50					132.75		
29.00		1.0	4.50					130.50		
	28.75			2.50	11.25	7.0	31.50		323.44	905.62[5]
28.00		0.5	2.25					63.00		
27.50		0.5	2.25					61.87[5]		
27.00		1.0	4.50					121.50		
26.00	26.00	1.0	4.50	5.00	22.50	18.0	81.00	117.00	585.00	2.106.00
24.00		3.0	13.50					324.00		
	23.75			15.00	67.50	20.0	90.00		1.603.12[5]	2.137.50
22.00		5.0	22.50					495.00		
	21.50			20.00	90.00	15.0	67.50		1.935.00	1.451.25
21.00		7.0	31.50					661.50		
20.00		8.0	36.00					720.00		
19.50	19.50	10.0	45.00	25.00	112.50	15.0	67.50	877.50	2.193.75	1.316.25
19.00		20.0	90.00					1.710.00		
18.50		10.0	45.00					832.50		
18.00		10.0	45.00					810.00		
	17.75			20.83	93.73[5]	11.0	49.50		1.663.80	878.62[5]
17.50		8.0	36.00					630.00		
17.00		3.0	13.50					229.50		
16.50		4.0	18.00					297.00		
16.00	16.00	5.0	22.50	9.17	41.26[5]	9.0	40.50	360.00	660.24	648.00
								8.863.87[5]	9.333.68	10.172.25

V. THE LOGNORMAL DISTRIBUTION USED AS A MARKET-ANALYTICAL INSTRUMENT IN SUCCESSIVE MARKET SITUATIONS

1. Price-quantity developments for individual products

In an article by S. Bennett and J.B. Wilkinson[14] a detailed discussion is to be found on the developments in the price-quantity relationships in 'in-store' experiments.
The prices of a number of selected products were weekly changed by a certain percentage over a period of six weeks. The actual prices of these products for each week were published in a local paper by means of advertisements. The experiment was carried out in a discount-store forming part of a regional chain.
The 5 products involved were: sweets (M & M candy), motor oil (Quaker State), aspirins (Bayer), flashcubes (Sylvania) and a universal cleaning agent (Janitor). The results of this experiment are stated below in respect of 3 of these 5 products, i.e. Quaker State motor oil, Sylvania flashcubes and the Janitor cleaning agent).

Quaker State motor oil (weekly price reduction 15%, normal selling price $ 0.53)

Price in week	Price in dollars ($)	Quantity	Percentage	Cumulative Percentage
1	0.73	26	1.17	1.17
2	0.62	70	3.15	4.32
3	0.53+	216	9.73	14.05
4	0.45	305	13.73	27.78
5	0.38	871	39.22	67.00
6	0.32	733	33.00	100.00
		2,221	100.00	

\+ = normal selling price

Sylvania flashcubes (weekly price reduction 10%, normal selling price $ 1.49)

Price in week	Price in dollars ($)	Quantity	Percentage	Cumulative percentage
6	1.84	4	4.44	4.44
5	1.66	11	12.22	16.66
4	1.49+	13	14.44	31.30
3	1.34	14	15.56	46.66
2	1.21	23	25.56	72.22
1	1.09	25	27.78	100.00
		90	100.00	

\+ = normal selling price

Janitor cleaning agent (weekly price reduction 10%, normal selling price $ 0.77)

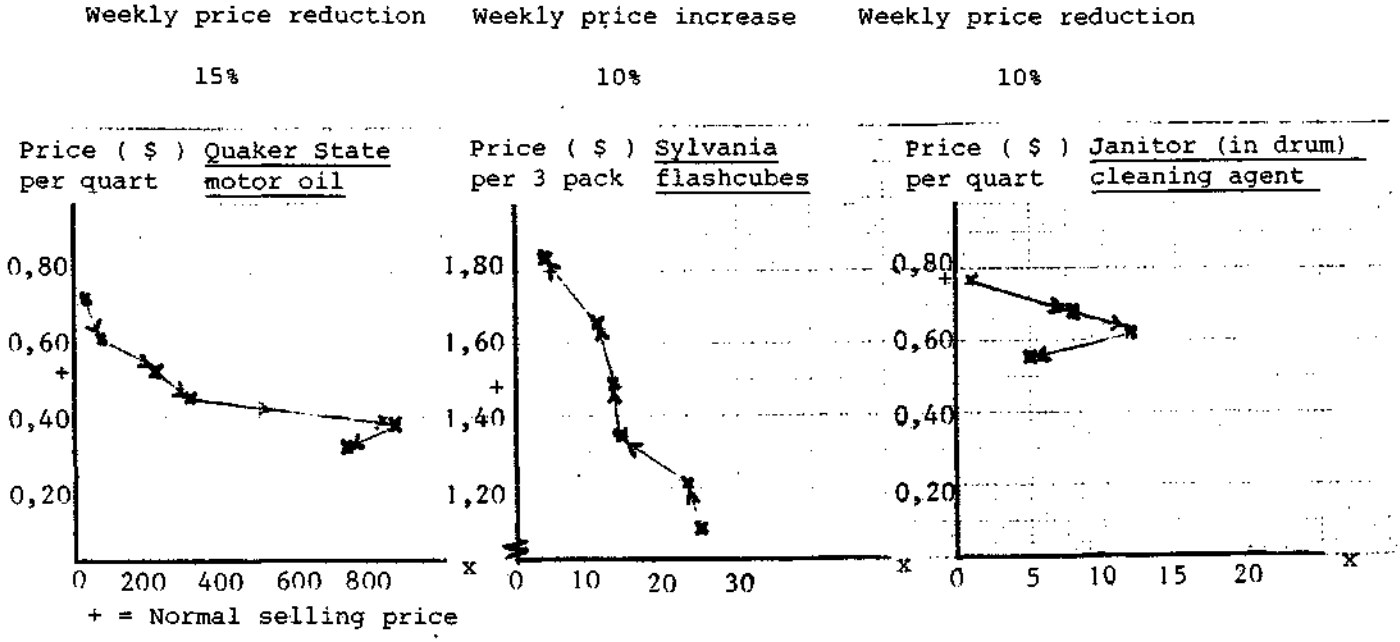
Price in week	Price in dollars($)	Quantity	Percentage	Cumulative percentage
1	0.95	--	--	--
2	0.85	--	--	--
3	0.77+	1	3.85	3.85
4	0.69	8	30.77	34.62
5	0.62	12	46.15	80.77
6	0.56	5	19.23	100.00
		26	100.00	

+ = normal selling price

For M. & M candy the price, starting at $ 0.31, was weekly increased by 15% to $ 0.71 while the price for Bayer aspirins was reduced by 10% per week from $ 1.02 to $ 0.61. The prices of the competing products were kept unchanged during the whole of the experiment. At the lowest prices for Quaker State motor oil and M & M candy a huge **increase in sales** of the competing products took place (i.e. Penzoil and Herseyettes respectively). Both in the case of Sylvania flashcubes and Bayer aspirins it was found that the sales of the competing products remained virtually unchanged in spite of the considerable price changes. The sales of competing cleaning agents during the experimental period remained more or less constant with the exception of the week in which Janitor was sold at its normal selling price. In this week the sales of the competing product dropped (Formula 409).

A graphic representation of the determined price-quantity combinations plotted on ordinary graph paper shows the following picture for the three products concerned.

FIG. 7



The two investigators, Bennett and Wilkinson, regarded these results as supporting the research results obtained by Tull and Pessemier, who had drawn up similar 'demand curves' that were characterised by a positive slope, kinks and sudden backward bends. They therefore hold the opinion that 'short-term' *) demand-curves occurring in realistic conditions, i.e. actual buying situations, do not show the same regularly declining course as is assumed in the price theory. Their conclusion is that demand curves in real buying situations show an irregular and discontinuous course and even show backward bendings, which are known to occur in the case of inferior goods.
In the study undertaken by Tull and Pessemier the kink of the demand curve was usually found to occur near the normal price. In the former investigation the 'kinks' were observed at prices other than the regular selling price. But in the case of Pessemier the persons taking part in the experiment were aware of the normal selling price, which was not the case in the former study. According to Bennett and Wilkinson these kinks represented a price that was of phychological significance to the buyers. The positive slopes and regressions in the curves were, in their opinion, 'difficult to interpret' and they held that the cause had to be explained by 'uneconomic' buying behaviour or market saturation. The conclusion of these scientists is therefore that the only thing that can be said with certainty is that price reductions (within certain limits) result in increased sales whereas price increases result in decreased sales. On the basis of the results of their research they conclude that it is hardly possible to prove the existence of a precise, functional and systematic relationship between prices and quantities sold.

However, if we approach the results of the investigation by Bennett and Wilkinson by means of the lognormal method, the results obtained are different. The tables can now be considered as showing the distribution of the quantities of products bought at particular prices. In that case we are dealing with a frequency distribution (density function cf. Fig. 1; the skew Gauss distribution) in which the distribution of the numbers of transactions closed at the various prices is shown in the given situation.
Although in this experiment efforts were made to create the circumstances in which the application of the 'ceteris paribus' condition, i.e. amongst other things, keeping the prices of competing products constant, was justified, the assumed constant nature of the buyers' preference during the (6-week) period of the experiment remains an open question and one that has not been explicitly justified by the investigators.
However, when the normal calculation according to the lognormal method is carried out, i.e. converting the quantities into percentages which are then cumulated, and these price/quantity combinations are plotted on lognormal graph paper (Fig. 8), it turns out that a virtually lognormally distributed course of the demand is obtained in all cases.

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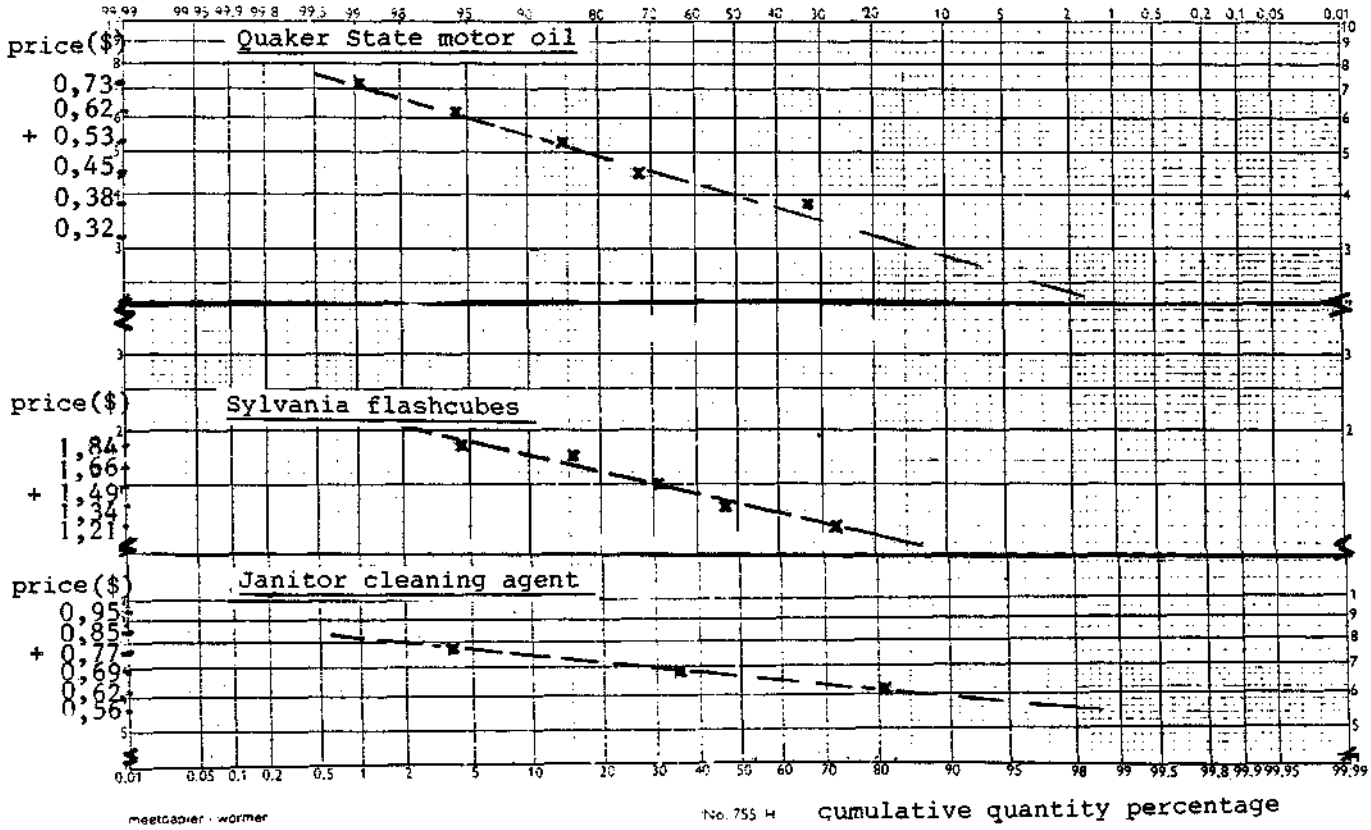
*)The short-term period mentioned in the article by Bennett and Wilkinson corresponds with a period that is approximately the same as the 'sales-promotions-planning' period for small retailers. In this sense this short-run period differs from the point-in-time conception in the classical demand theory and the long-run statistical demand analysis.

FIG. 8

Testing the lognormally distributed course of the demand for 3 products



In view of the above lognormal distribution it may be said that there is one demand curve (cf. p.6) for each of these products.
Each lognormal distribution represents, in fact, the demand curve of a heterogeneous group of buyers in which the revealed preferences of the buyers is expressed in the quantity of products valued at a given price or higher. To assume that the subjective valuations are constant during this experimental period would therefore appear to be justified.

The backward bends of the curve that were observed and considered difficult to explain by Bennett and Wilkinson can now be explained very easily.
We are here dealing with the kink of the skew Gauss distribution, below which the density function (marginal demand) declines. The total demand increases relatively less beyond the starting point of this backward bend.
A remarkable fact is, however, that in the material produced by Bennett c.s. these backward bends are only observed in the case of price reductions. Apparently the backward bending occurs in the case of motor oil and the cleaning agent as a result of the price reduction whereas in the case of the flashcubes the course of the curve remains above this point on account of the price increase.
Also the actual market saturation that occurs in motor oil and M & M candy at the lowest price simultaneous with a huge increase in the sales of competing products (cf. p. 22), is now simple to explain.

A market saturation of this kind for one product/variety, should by no means be confused with market saturation as for a product group as a whole. In the latter case, the buyers hold such large stocks of the product concerned that a price reduction no longer results in increased sales. The overall market for this product group is then saturated. In this case, the saturation only relates to an individual product from a product group in which the sales of competing products remain the same or may even show a considerable rise (cf. p. 22). This kind of market saturation affecting an individual product is related to a reduced confidence among buyers in the quality-price relationship of the products concerned. This agrees with the findings of other investigators, amongst others, De Koning[15] and Kuhlmeijer[16], who both postulate that buyers operate between certain price limits which differ for each separate product. At higher prices the product is considered 'too expensive' which means that the price acts as a restrictive factor on the demand. At low prices the buyers show a reduced confidence in the quality of the product (at that price) which means that the quality acts as a restrictive factor on the demand. This explains why a reduced demand also occurs at lower prices (cf. pp. 9, 10, market saturation and inferiority of goods).
Contrary to the explanations advanced by Bennett and Wilkinson for this phenomenon of individual market saturation for a product, the application of the lognormal distribution method shows that there is no question whatsoever of 'uneconomic buying behaviour' or (total) market saturation covering an entire product group.
Also Monroe submits that buyers operate within certain price limits, stating at the same time that the phenomenon of decreased sales at lower prices remains entirely undiscussed in the economic theory of consumer behaviour[17].
Generally speaking, market saturation at large is only dealt with in economic theories as a phenomenon occurring for product groups as a whole whilst inferiority of goods is treated as an exceptional example, i.e. applying to a few goods only.
By using the theory of the lognormal distribution it is shown that market saturation may occur in each separate product if the buyers no longer have confidence in the price-quality relationship of the product in question. At a particular (low) price the product is looked upon as being inferior. There are price limits for each individual product/brand. Above these limits the buyers consider a product too expensive and below the limit they regard it as inferior. Empirical studies have proved this thesis.

The example proves that the lognormal method provides a good description of the empirical relationship between prices and demanded quantities. However, the lognormal distribution solely indicates the relationship between the price level and the percentage of the market volume. For the moment it is still impossible to make any statements on the absolute market volume.

2. Price-quantity developments in successive market situations for product groups

The lognormal distribution theory has also been used to carry out a number of comparative analyses of successive market situations for various product groups within the scope of this study.
As an example of an analysis of this kind we have chosen the market for books in The Netherlands during the years 1979, 1980 and 1981, since the sales of books showed a particularly strong decline during this period.
The figures for this analysis were obtained from Attwood B.V., Dongen, The Netherlands.

These figures show that the distribution of the price-quantity combinations changed during these years, which is illustrated by the following analysis of the book market for these years.

Number of books bought per 100 households (panel 10,000)

Total book market	1979			1980			1981		
price in Dfls.	number	%	cum.%	number	%	cum.%	number	%	cum.%
50.- and over	5.9	1.8	1.8	5.1	1.6	1.6	2.4	1.3	1.3
30.- to 50.-	15.6	4.6	6.4	15.0	4.7	6.3	9.8	5.4	6.7
20.- to 30.-	42.3	12.6	19.0	38.5	12.2	18.5	22.5	12.5	19.2
10.- to 20.-	94.3	28.1	47.1	79.9	25.3	43.8	42.8	23.7	42.9
4.50 to 10.-	80.6	24.0	71.1	92.4	29.2	73.0	53.2	29.5	72.4
Less than 4.50	97.4	29.0	100.0	85.0	26.9	99.9	49.8	27.6	100.0
	336.1	100.0		315.9	99.9		180.5	100.0	
Total amount spent	Dfls.4,420.-			Dfls.4,025.-			Dfls.2,302.-		
Average price	Dfls.	13.15		Dfls.	12.74		Dfls.	12.75	

The figures show that the number of books sold considerably declined within a brief period of time, i.e. 1980 - 1981, compared with the starting year, 1979. Although there are quite a few objections to be raised to the figures obtained via the Attwood panel (different price classes, bad distribution of the price classes throughout the price range), the figures still offer an opportunity of carrying out a more detailed analysis of the book market by using the lognormal distribution method.
There is clear evidence to show that the absolute number of books sold in each price class drastically declined in 1980, with the exception of the books sold in the price class Dfls. 4.50 to Dfls. 10.- (i.e. which rose from 80.6 to 92.4). However, the distributions on a percentage basis of the number of books in the various price classes display considerable shifts. In some price classes there was a declining trend (e.g. Dfls. 50.-/up from 1.8% to 1.6% and 1.3%, Dfls. 10.- - Dfls. 20.- from 28.1% to 25.3% and 23.7%) whereas in the other price classes an earlier decline was followed by an increase (i.e. Dfls. 20.- - Dfls. 30.- from 12.6% to 12.2% and then 12.5%, below Dfls. 4.50 from 29.0% to 26.9% and then to 27.6%).

Such irregular shifts in the percentage distribution of the numbers of books sold in the various price classes are an indication of considerable changes having occurred in the buyers' subjective valuations. When the cumulative quantity percentages of the books sold in the years 1979, 1980 and 1981 are plotted on lognormal graph paper they indicate that the initially almost lognormally distributed price-quantity combinations (i.e. in 1979) display a worsening break in the following years, i.e. in 1980 and 1981. Although the incidence of this break can already be observed in 1979, the break becomes increasingly evident in the years 1980 and 1981.
Based on the first 4 observation points in 1979 a normline can be drawn that may be considered as being the most likely demand curve without the incidence of important, systematic or non-random factors (--- in Fig. 9.1.). It is clear that particularly the last observation point (i.e. Dfls. 4.50 - Dfls. 10.-; 71.1%) shows a distinct deviation from this normline.
Based on the normline it may be said that a valuation of Dfls. 4.50/up should apply to approx. 83% of the market or that for a market volume of 71.1% a valuation of Dfls. 6.- and more should apply. It would seem that it should be possible to increase the prices within this price class, or, alternatively, to increase the market share covered by this price class by approx. 10%.

FIG. 9.1.

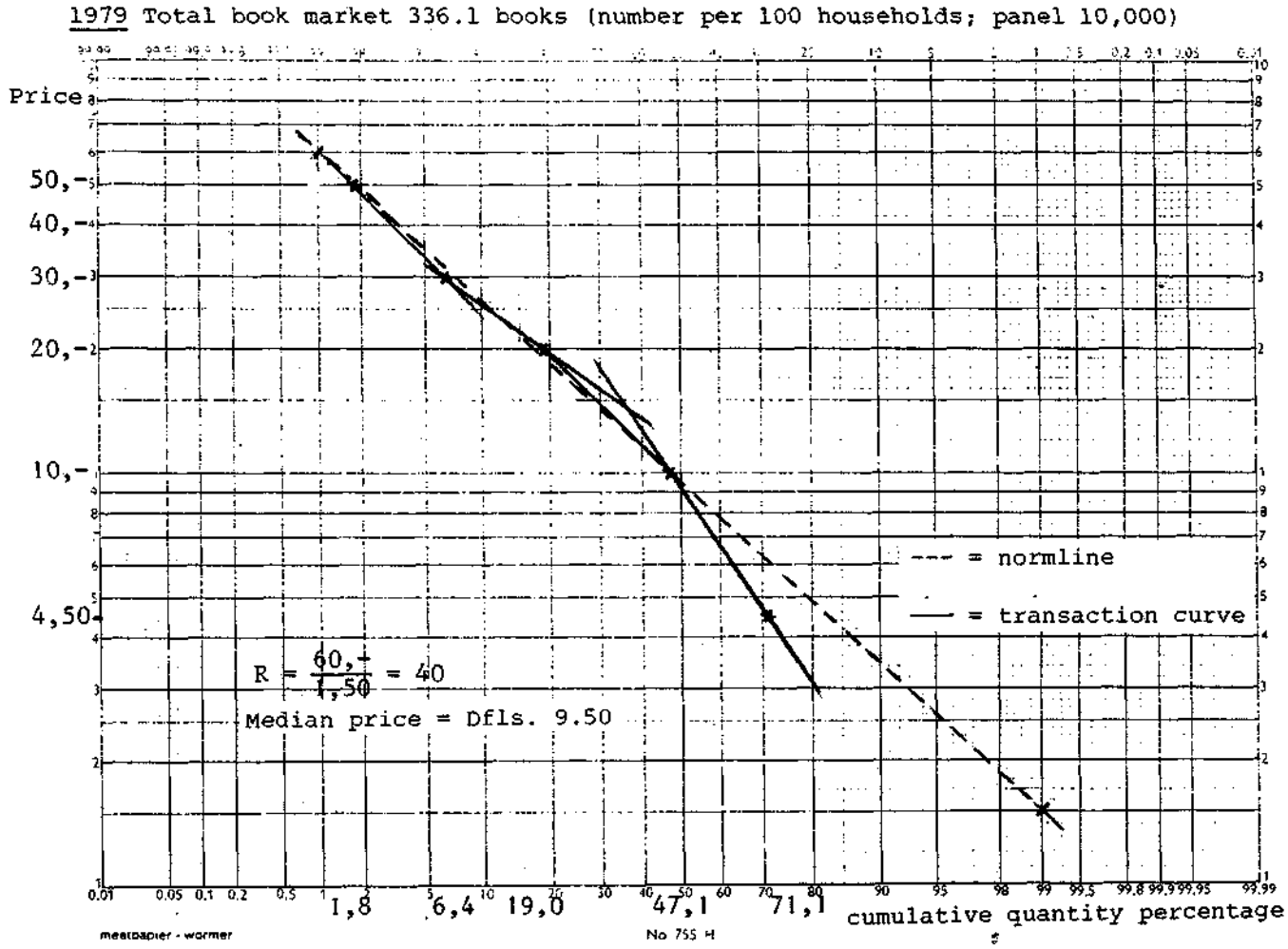


Fig. 9.2.

1980 Total book market 315.9 books (number per 100 households; panel 10,000)

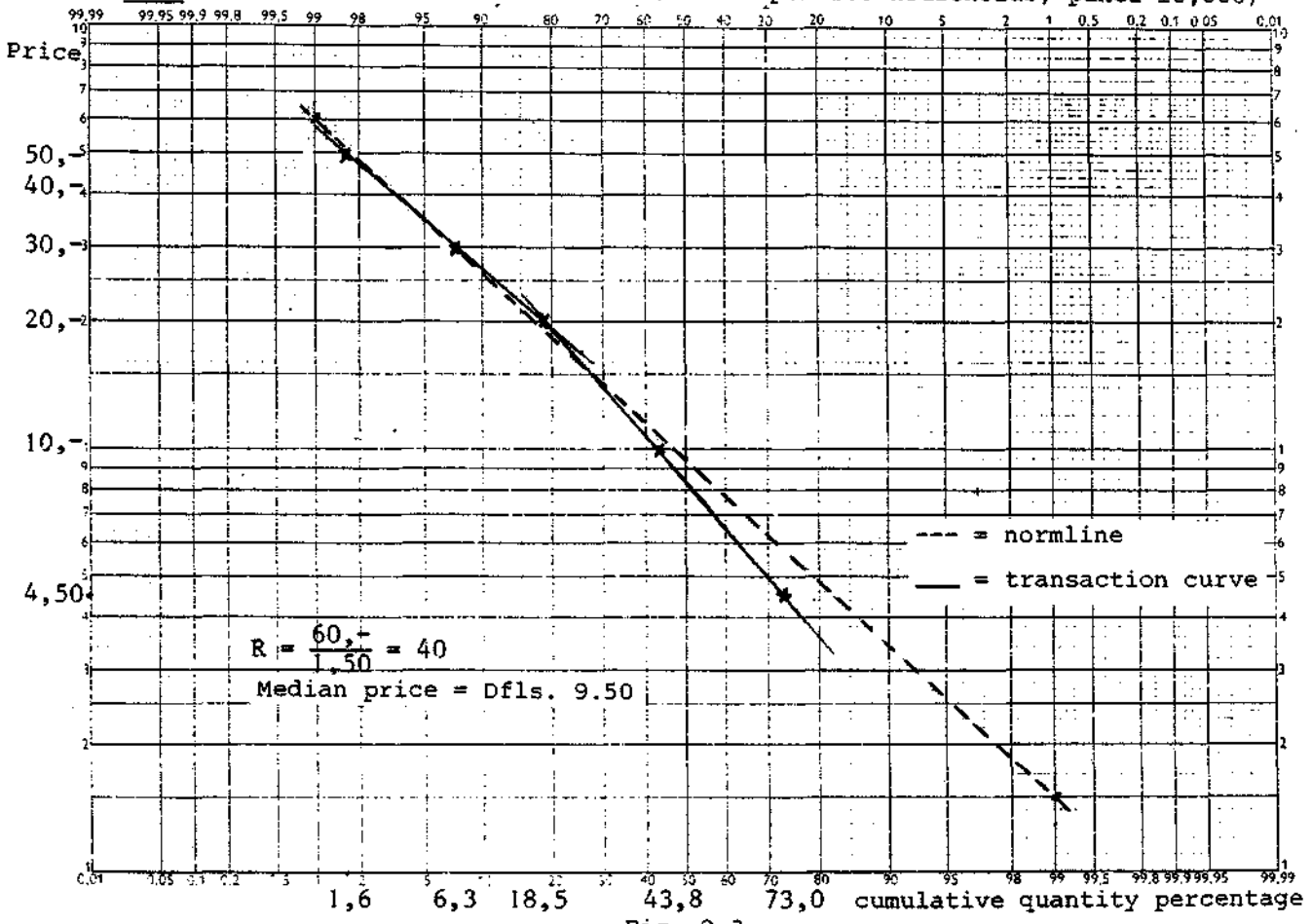
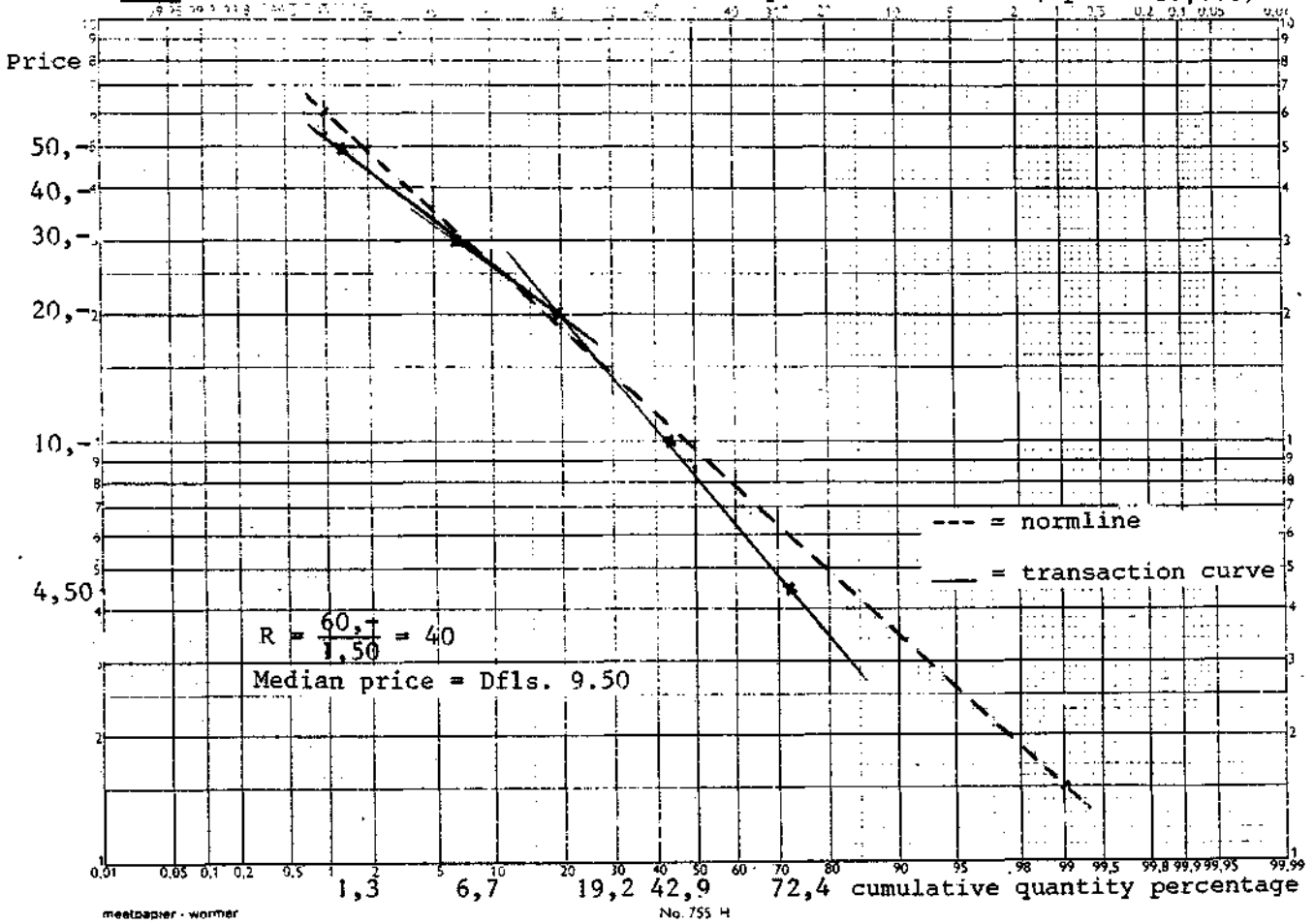


Fig. 9.3.

1981 Total book market 180.5 books (number per 100 households; panel 10,000)



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The normline constructed for 1979 may also serve as indicating the reference limits for the market analysis in the years 1980 and 1981. This normline is determined by the range (R) of 40 and the median price (M) of Dfls. 9.50.
If the deviations from this normline over the years 1979, 1980 and 1981 are considered more carefully, a break in the buyers' valuation pattern, especially in 1980 and 1981, can be observed. The transaction curves for 1980 and 1981 actually consist of two separate lognormal distributions. As mentioned before, the existing incidence of this break can already be observed in 1979. The transaction curve has a practically lognormal distribution in 1979, however. The small deviations at Dfls. 20.- and Dfls. 30.- (i.e. above and below the normline respectively) are irrelevant for the present purpose, since they are not sufficiently conspicuous and may be seen as forming part of the lognormal distribution. Only the deviation at Dfls. 4.50 requires a further explanation.
In view of the pattern of the transaction curves in 1980 and 1981 it is highly plausible that an explanation for this deviation in 1979 has to be looked for in the latent break in the valuation pattern shown by the buyers. The cause of this break in the buyers' valuation pattern is to be found in important factors which may originate from:

- the supply
  In this case the differences in the composition and handling of the market instruments by the producers (producers' mix) are so great that the buyers do not view the total product assortment as a single undivided product group. A similar situation occurred in the analysis of the margarine market. Although margarines and halvarines had, in principle, been classed as one product group, the break occurring in the transaction curve indicated that buyers clearly distinguished between margarine as one product group and halvarine as another, separate product group (i.e. a considerable substitution discontinuity).
- the distribution channels
  The differences between the retail links (retail-mix) may be so large that buyers make a clear distinction between the various types(kinds) of sales outlets. Since the characteristics of the retail links in the buyers' judgment are also counted as part of the characteristics of the transaction, such differences among the retail links may also cause a break in the valuation pattern. Also in this instance the buyers actually view the group of combined products as constituting more than one product group.
- the buyers/consumers
  The composition of the buyers' group may show different kinds of characteristics so that there are two or more different (sub-) groups of buyers. When various buyer groups are combined, this may also result in one or more breaks occurring in the total distribution.

- External factors
  External factors may exert an equally or unequally distributed influence on the buyers' behaviour, product supply and producers' behaviour as well as the number and variety of retail links.
  In these cases changes may occur in the total number of transactions as well as the number of transactions closed at the various prices.

As far as the book market is concerned, it may be said that during the period 1979 through 1981 no notable changes occurred in the product supply (i.e. the volume and composition of the assortment) and the retail links (i.e. the number and types of sales links).
The only remaining causes to explain the incidence of the break are the buyers group and the external factors.
It would seem likely to assume that (the fear of) the general decline in the (discretionary) purchasing power was the cause of the considerable decline in the number of books sold. This drop in the sales figures occurs for the first time after the third quarter of 1980 increasing progressively in 1981. (It should be noted that the original figures were split up into quarterly data and are here presented in the annual totals which are considered to be sufficiently illustrative). Although a loss in purchasing power and the attendant changes in the buyers' spending pattern results in some instances in maintained or even expanded spending for certain product groups, the conclusion seems justified that considerably less is spent on the purchasing of books.
Such a general decline in purchasing power does not have the same effect on all buyers. In the book market a shift can, on the one hand, be observed in the number of books sold in the various price classes while, on the other hand, considerably reduced sales occurred in all price classes (i.e. buyers disappear or buy considerably less).
As a result of these developments the break in the transaction curve emerges clearly. Mainly because of the decline in purchasing power, the existence of two different groups of buyers becomes evident. This means that the book market actually consists of two sub-markets in which a distinction can be made between a market for books priced at over Dfls. 20.- and another below this price.
Since it was impossible within the scope of this study to make a further analysis of the differences (in buying behaviour, etc.) between these two buyer groups, only the possible causes can be suggested. Further research might, for instance, show that a distinction should be made between books bought for various purposes (e.g. for one's own use or as presents to be given to others) or various other uses (e.g. as works of reference, one-time reading or supplementing one's library).

Also considerable differences in the finish and type of product (e.g. hardback, paperback) might cause such different valuation patterns among buyers.

The fact that this break in the valuation pattern of the buyers in respect of the book market was found to exist, means that there is every reason to undertake a further investigation into this market and the buyers concerned. Such an investigation should be able to produce an explanation for the deviating buying behaviour shown.
Any characteristic differences in the changes in the subjective valuations of groups of buyers in these various (sub-)markets for books should then form the basis of future policies to be followed by producers and retailers of books.
These policies might result in a future splitting up of the book market into two (clearly distinguished) sub-markets or influencing the (potential) buyers by using market instruments (both producers and distributors) so as to avoid the incidence of breaks occurring in the valuation patterns. The result would be that only one single market for books would exist. As mentioned earlier, the scope of this investigation made it impossible to carry out a thorough study into the characteristics of the book market. The only conclusion must be that a study of this kind is needed to gain an improved insight into this market and to promote an efficient and effective use of the marketing instruments.

In the analyses carried out in this study into the price-quantity relationships in successive market situations for certain product groups three developments have come to the fore i.e.

1) When fairly general price changes occur for all the products within a product group (e.g. on account of changed raw material prices), as happened in the motorcar industry during a succession of years and in the coffee prices between 1976 and 1981, a virtually parallel shift occurs in their original lognormal distribution. This means that the range of the distribution (= price at 1% of the market volume : price at 99% of the market volume), remains virtually unchanged whereas the median price (= price at 50% of the market volume) undergoes a change. In these instances the market may be said to be fairly stable and that shifts only occur in the market proportions held by the various products in the product group in question (i.e. shifts in their respective market shares). The parallel shifts in the lognormal distribution were seen to be generally smaller (i.e. the percentage differences in the median prices) than had been expected on account of the general percentage changes of the price levels.

This is caused by the change-over from relatively expensive products to relatively cheap products during a general price increase and vice versa during a price decrease. However, this phenomenon does not immediately occur at each price change. There is a fairly high brand loyalty among buyers. As long as such a change remains within certain limits buyers will remain loyal to their brand. Only when the limit is exceeded will any change to another product be made, depending on the extent of buyers' brand loyalty. This explains why the total percentage of a price change does not result in the same percentage change in the median prices.

2) If no notable price changes occur during a prolonged period for products falling within the same product group the lognormal distribution remains more or less constant (i.e. showing the same range and median price). It is quite possible for shifts to occur in the proportional market shares. This change in the market shares held by separate products/brands of a product group are caused by proportional changes in the composition and use of the marketing mix by the various producers and/or retail links.
   A similar situation occurred in the market for detergents in The Netherlands. In 1978 this market was virtually lognormally distributed. Also in 1981 the distribution proved to be virtually lognormal showing the same range and median price as in 1978. However, shifts proved to have changed the proportional market shares held by the various products/brands of detergents.

From these two examples it is clear that the proportional subjective valuations of the buyers remain fairly constant (i.e. showing the same range) during a prolonged period of time. Depending on the extent of the general price change occurring in an entire product group the median price shifts to a different level. In some situations, however, the proportional subjective buyers' valuations do not continue to show this stability.

3) When great changes occur in the proportional composition and use of the marketing-mix for the separate products in a product group and/or changes in the (discretionary) purchasing power occur which are not equally proportional in their effect on all buyers, they result in entirely different distributions (i.e. showing a different range and/or median price) or breaks in the distribution. This is the kind of situation which occurred in the book market in the years 1979 - 1981.

VI. PROVISIONAL CONCLUSIONS OF THE STUDY INTO THE USE OF THE LOGNORMAL DISTRIBUTION AS A MARKET-ANALYTICAL INSTRUMENT

This report contains the provisional results of a study into the possibilities of developing an instrument to analyse the demand for marketing purposes. It was based on practical examples forming a maximum illustration of the potential use of such instruments.
This study was primarily aimed at analysing the price-quantity relationship in the case of the simultaneous or successive occurrence of price differences in consumer goods in incompletely separated sub-markets (i.e various distribution channels).
In the tradional theories attention is also paid to the occurrence of price differentiation c.q. price discrimination, in which an actual separation of the various sub-markets is assumed. However, these theories do not indicate a possible relation between the various forms of price differentation or price discrimination according to size and nature. The theory of the lognormal distribution of the demand shows definite evidence of the existence of such a relation based on the subjective valuation patterns of buyers and that, moreover, the existence (in incompletely separate markets) can also be shown to exist by means of empirical research.
The inferiority of goods is characterised as an exceptional case in the more traditional price theory. Only a limited number of goods are considered to be inferior in certain circumstances.
According to the lognormal distribution theory, each product when reduced in price below a certain level, is viewed by the consumers as an inferior product (i.e. loss of confidence in the quality-price relationship) resulting in an actual market saturation for the product concerned.
According to Monroe this form of market saturation for a separate product is not explictily recognised by the traditional theory of consumer behaviour[17].
In the traditional theory of consumer behaviour the conception of market saturation relates particularly to entire product groups, i.e. all buyers are holding sufficient stocks of these (kinds of) products. The lognormal distribution theory shows evidence that market saturation may occur in respect of each separate product. In empirical researches this form of market saturation was in some instances shown to be accompanied by unchanged, or even increased sales of competing products.

The provisional results of this investigation are as follows:
- Based on the plausibility test - testing the lognormal course of a distribution by means of lognormal graph paper - the assumption that each transaction is closed under the influence of a great number of individually unimportant factors would seem to be justified for the moment.

- So far it has been possible to explain any deviations by the incidence of one or more major factors.
- The changes in the subjective valuations, inasmuch as they are expressed in the prices paid and the quantities of goods bought, by one large heterogeneous group of buyers displaying no characteristic differences, are lognormal and remain stable for a prolonged period of time, provided that no significant influences occur.
- The theory of the lognormal distribution method can therefore be used as a market-analytical instrument to detect 'imbalances' in a market. These 'imbalances' can simply be demonstrated and made visible with the help of lognormal graph paper.
- With due allowance for the anticipated lognormally distributed demand pattern, recommendations can then be made in order to eliminate such 'imbalances' inasmuch as they are responsible for less optimal/efficient trading results.
- The operating sphere of the lognormal method extends to the product assortment (composition and proportional relations between the products produced), the total assortment in a particular market (product group) and the distribution and quality differences among the various sales outlets.
- A comparison of successive market situations for separate products or product groups may yield indications on the anticipated future trend of the demand.
- However, the lognormal method can only be used to provide indications on the percentage distribution of the demand in the various price classes. For the moment no predictions can be made in respect of the absolute market volume.

A continued study wil be required to extend the plausibility test with regard to the anticipated lognormally distributed pattern of the demand and will have to cover more categories of goods (both commodities and durable consumer goods and services)
At the same time a detailed study should take place into the remaining demand relationships (e.g. the quality-demand, distribution-demand, advertising/promotion-demand relationships etc.) while the results of the investigation into the price-demand relationship will have to be worked out in further detail.

Summarising, it would appear that based on the provisional results of this study the lognormal distribution theory is a useful demand-analytical instrument for marketing purposes .

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